International Journal of Theoretical Physics, Vol. 39, No. 6, 2000

Two-Mode Normal Squeezing of a Nondegenerate Bimodal Multiquanta Jaynes–Cummings Model in the Presence of Stark Shifts

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Received December 15, 1999

We investigate the two-mode normal squeezing in the presence of Stark shifts for a generalized Jaynes–Cummings Model (JCM) with a two-level particle (atom or trapped ion) and two modes interacting nonlinearly. The effect of the relative phase between the atomic superposition state and the coherent field in the presence and absence of the Stark shift on squeezing is studied. Different values for the parameters of the atomic coherent state are taken.

1. INTRODUCTION

The Jaynes–Cummings Model (JCM) [1] of a two-level atom in interaction with a single mode of an electromagnetic field has been studied extensively [2–5]. Sukumar and Buck [6, 7] proposed two exactly solvable generalizations of the Jaynes–Cummings model, one involving intensity-dependent coupling and the other involving multiphoton interaction between the field and the atom. A number of generalized JCM models have been investigated [8–17]. Zhu and Scully [18] and Boone and Swain [19] studied the properties of nondegenerate and degenerate two-photon lasers. They found that the photon distribution, the linewidth, and the frequency shift depend strongly on the detailed atomic structure because of the effects of ac Stark shifts. Brune *et al.* [20] showed that the ac Stark shifts may be proposed to realize a quantum-nondemolition scheme to measure the number of photons stored in a high-**Q** cavity. Also, the quantum properties of a coupled atom–field system, such as the collapse and revival of atomic inversion [21–25],

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0020-7748/00/0600-1499$18.00/0

atomic transition lines [24], the squeezing of the field [25], and the phase properties of the field [26], can be changed drastically due to the influence of ac Stark shifts. A number of authors [27–34] have studied various dynamical aspects of the nondegenerate two-mode JCM. In particular, Cirac and Sanchez-Soto [35] pointed out that, in the degenerate two-photon Jaynes–Cummings model in the presence of Stark shifts, the field which traps the atom in the linear superposition of its two levels must be a single-mode squeezed vacuum field. In other studies, the particle (atom or trapped ion) is taken to be prepared initially in a coherent superposition of its upper and lower levels, and it interacts with a single coherent mode [36, 37].

In this article, we find the wave function in the presence of Stark shifts for the system of two modes in interaction with the particle. Then we calculate two-mode normal squeezing for different values of the parameters in the particle coherent state $|\Psi_{\text{particle}}(0)\rangle$ where the two modes are initially in coherent states. The effects of the change in the relative phases in the presence and absence of the Stark shift on the two-mode normal squeezing are studied. This article is organized as follows: In Section 2, we present the wave function for the nondegenerate bimodal multiquanta Jaynes–Cummings Hamiltonian. Section 3 is devoted to an investigation of the influence of the Stark shift on two-mode normal squeezing in the JCM either in the resonant and off-resonant. Concluding remarks are provided in the last section.

2. THE WAVE FUNCTION OF THE SYSTEM

We consider the nondegenerate bimodal multiquanta JCM with a detuning parameter. The nondegenerate bimodal multiquanta JCM consists of a two-level particle (atom or trapped ion) and two modes interacting nonlinearly. The interaction between the particle and the field is affected by k_i quanta of the ith mode. The Hamiltonian for the system in the rotating wave approximation is written as

$$\hat{H} = \frac{\omega_0}{2}\hat{\sigma}_z + \sum_{j=1}^{2}\omega_j\hat{a}_j^\dagger\hat{a}_j + \hat{a}_1^\dagger\hat{a}_1\beta_1|g\rangle\langle g| + \hat{a}_2^\dagger\hat{a}_2\beta_2|e\rangle\langle e|$$
$$+ \lambda(\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2}\hat{\sigma}_- + \hat{a}_1^{k_1}\hat{a}_2^{k_1}\hat{\sigma}_+) = \hat{H}_0 + \hat{H}_{\text{int}} \quad (1)$$

This Hamiltonian can be generated from a Raman coupling for an effective three-level ion in a Λ-configuration confined with a two-dimensional harmonic trap as described in ref. 38.

An ion confined in an electromagnetic trap can be regarded as a particle with quantized center-of-mass (c.m.) motion moving in a harmonic potential. A classical laser driving field changes the external states of the ion motion by exciting or deexciting the internal atomic states of the trapped ion. After using the adiabatic elimination procedure, a general form of this Hamiltonian is obtained. If both the vibrational amplitudes of the ion are much smaller than the laser wavelength, then the Lamb–Dicke limit can be used. In this limit one has only the leading term in the Lamb–Dicke parameter η whose square gives the ratio between the single-photon recoil energy to the energy level spacing in the harmonic oscillator potential. This model, Eq. (1), can be obtained in the Lamb–Dicke approximation and in the limit of suitable trap anisotropy and specific sideband detunings of the laser. In this case the $\hat{a}$'s describe vibrational modes and $\hat{\sigma}$'s describe the ion internal states. This Hamiltonian generalizes that of ref. 39, where one of the $\hat{a}$'s describes the cavity mode and the other describes the vibrational mode of the ion in the cavity QED of a trapped ion. As the coupling between the vibrational modes and the external environments is extremely weak, dissipative effects, which are inevitable from cavity damping in the optical regime, can be significantly suppressed for the ion motion.

It is easy to prove that $\hat{H}_0$ and $\hat{H}_{\text{int}}$ commute i.e.,

$$[\hat{H}_0, \hat{H}_{\text{int}}] = 0 \tag{2}$$

where

$$\hat{H}_0 = \omega_1\left[\hat{n}_1 + \frac{k_1}{2}(\hat{\sigma}_z + \mathrm{I})\right] + \omega_2\left[\hat{n}_2 + \frac{k_2}{2}(\hat{\sigma}_z + I)\right] \tag{3}$$

and

$$\begin{aligned}\hat{H}_{\text{int}} = {} & \frac{\Delta}{2}\hat{\sigma}_z + \hat{a}_1^{\dagger}\hat{a}_1\beta_1|g\rangle\langle g| + \hat{a}_2^{\dagger}\hat{a}_2\beta_2|e\rangle\langle e| \\ & + \lambda(\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2}\hat{\sigma}_- + \hat{a}_1^{k_1}\hat{a}_2^{k_2}\hat{\sigma}_+)\end{aligned} \tag{4}$$

with the detuning parameter Δ given by

$$\Delta = (\omega_0 - k_1\omega_1 - k_2\omega_2) \tag{5}$$

where $\hat{a}_j(\hat{a}_j^{\dagger})$ and $\hat{n}_j = \hat{a}_j^{\dagger}\hat{a}_j$ are the annihilation (creation) and number operators for the jth mode, λ is the particle–field coupling constant, β_1 and β_2 are parameters describing the intensity-dependent Stark shifts of the two levels that are due to the virtual transition to the intermediate relay level; ω_1 and ω_2 are the field frequencies for the two modes, ω_0 is the transition frequency of the particle (atom or trapped ion), $\hat{\sigma}_z$ is the population inversion operator, and $\hat{\sigma}_\pm$ are the "spin-flip" operators, which satisfy the relations $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$ and $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$. When $k_1 = k_2 = 1$, Eq. (1) reduces to that of ref. 28.

Let us consider the particle prior to the interaction to be prepared in a coherent superposition of its excited and ground states [36, 37],

$$|\Psi_{\text{particle}}(t = 0)\rangle = \cos\left(\frac{\theta}{2}\right)|e\rangle + e^{-i\phi}\sin\left(\frac{\theta}{2}\right)|g\rangle \tag{6}$$

The initial particle–field state is then a product of the superposition state and the field in a photon coherent state or the squeezed field

$$|\Psi(0)\rangle = \sum_{n_1,n_2} q_{n_1}q_{n_1}|n_1, n_2\rangle|\Psi_{\text{particle}}(t = 0)\rangle \tag{7}$$

where

$$q_{n_j} = e^{-\bar{n}_j/2}\sqrt{\bar{n}_j^{n_j}/n_j!} \qquad (j = 1, 2) \tag{8}$$

By using the interaction Hamiltonian (4) and the initial condition (7), we find the solution of the Schrödinger equation

$$i\frac{d}{dt}|\Psi(t)\rangle = \hat{H}_{\text{int}}|\Psi(t)\rangle \tag{9}$$

in the form

$$\begin{aligned}|\Psi(t)\rangle = \sum_{n_1,n_2} q_{n_1}q_{n_2}\Big\{&\exp[-i\lambda t\gamma_{n_1+k_1,n_2+k_2}]\cos\left(\frac{\theta}{2}\right)[A_{n_1+k_1,n_2+k_2}|n_1, n_2; e\rangle\\ &- i\nu_{n_1+k_1,n_2+k_2}R_{n_1+k_1,n_2+k_2}|n_1 + k_1, n_2 + k_2; g\rangle]\\ &+ \exp[-i\lambda t\gamma_{n_1,n_2}]e^{-i\phi}\sin\left(\frac{\theta}{2}\right)\\ &\times [A^*_{n_1,n_2}|n_1, n_2; g\rangle - i\nu_{n_1,n_2}R_{n_1,n_2}|n_1 - k_1, n_2 - k_2; e\rangle]\Big\}\end{aligned} \tag{10}$$

where the coefficients A_{n_1,n_2}, ν_{n_1,n_2}, and R_{n_1,n_2} are given by

$$A_{n_1,n_2} = \cos\lambda t\beta_{n_1,n_2} - i\left[\delta_{n_1,n_2} + \frac{\Delta}{2\lambda}\right]R_{n_1,n_2}, \qquad R_{n_1,n_2} = \frac{\sin\lambda t\beta_{n_1,n_2}}{\beta_{n_1,n_2}} \tag{11}$$

$$\beta^2_{n_1,n_2} = \left[\delta_{n_1,n_2} + \frac{\Delta}{2\lambda}\right]^2 + \hat{\nu}^2_{n_1,n_2}, \qquad \hat{\nu}^2_{n_1,n_2} = \frac{\hat{n}_1!}{(\hat{n}_1 - k_1)}\frac{\hat{n}_2!}{(\hat{n}_2 - k_2)!} \tag{12}$$

with

$$\gamma_{n_1,n_2} = \frac{\beta_2}{\lambda}(n_2 - k_2) - \delta_{n_1,n_2}, \qquad \delta_{n_1,n_2} = \frac{1}{2}\left[\frac{\beta_2}{\lambda}(n_2 - k_2) - \frac{\beta_1}{\lambda}n_1\right] \tag{13}$$

It is concluded that the former studies can be considered as special cases here; for example, by putting $k_1 = k_2 = 1$, $\beta_1 = \beta_2 = 0$, and $\Delta = 0$ we get the model studied by Abdel-Hafez [40]. The model of Nasreen [28] is obtained by putting $k_1 = k_2 = 1$. Therefore, this model represents a more generalized JCM than those studied before.

It must be noted that the state $|n_1, n_2; e\rangle$ means that the first mode is in the n_1th Fock state, the second mode is in the n_2th Fock state, while the third subscript stands for the excited particle state. Thus the expectation value of any operator Q and its dependence on time can be obtained through the formula (10),

$$\langle Q(t)\rangle = \langle \Psi(t)|Q|\Psi(t)\rangle \tag{14}$$

3. TWO-MODE NORMAL SQUEEZING OF THE FIELD

Now we study the two-mode normal squeezing of the field of the nondegenerate bimodal multiquanta JCM and discuss effects of the Stark shifts on this type of squeezing. The operators of the real parts of the quadrature are defined by [41]

$$\hat{Z}_1(t) = \frac{1}{2\sqrt{2}}[\hat{A}_1 + \hat{A}_1^\dagger + \hat{A}_2 + A_2^\dagger],$$

$$\hat{Z}_2(t) = \frac{1}{2i\sqrt{2}}[\hat{A}_1 - \hat{A}_1^\dagger + \hat{A}_2 - \hat{A}_2^\dagger] \tag{15}$$

where $\hat{A}_j = \hat{a}_j e^{i\omega_j t}$ and $\hat{A}_j^\dagger = \hat{a}_j^\dagger e^{-i\omega_j t}$ $(j = 1, 2)$ are slowly varying operators.

These operators satisfy the commutation relation

$$[\hat{Z}_1, \hat{Z}_2] = i/2 \tag{16}$$

and the uncertainty relation

$$(\Delta\hat{Z}_1)^2(\Delta\hat{Z}_2)^2 \geq 1/16 \tag{17}$$

The state of the field is said to be squeezed whenever one of the two quadratures $\hat{Z}_1$ and $\hat{Z}_2$ satisfies the relation

$$(\Delta\hat{Z}_1)^2 \text{ or } (\Delta\hat{Z}_2)^2 < 1/4 \tag{18}$$

On the other hand, the condition (18) can be rewritten as

$$W_j = (\Delta\hat{Z}_j)^2 - \frac{1}{4} \qquad (j = 1 \text{ or } 2) \tag{19}$$

and squeezing occurs when W_1 or $W_2 < 0$. In terms of the annihilation and creation operators of the field, we readily find that

$$W_1 = \frac{1}{8}\{2\langle\hat{A}_1^\dagger\hat{A}_1\rangle + 2\langle\hat{A}_2^\dagger\hat{A}_2\rangle + \langle\hat{A}_1^2\rangle + \langle\hat{A}_1^{\dagger 2}\rangle + \langle\hat{A}_2^2\rangle + \langle\hat{A}_2^{\dagger 2}\rangle + 2(\langle\hat{A}_1\hat{A}_2\rangle + \langle\hat{A}_1^\dagger\hat{A}_2^\dagger\rangle + \langle\hat{A}_1^\dagger\hat{A}_2\rangle + \langle\hat{A}_1\hat{A}_2^\dagger\rangle) - [(\langle\hat{A}_1\rangle + \langle\hat{A}_1^\dagger\rangle) + (\langle\hat{A}_2\rangle + \langle\hat{A}_2^\dagger\rangle)]^2\} \tag{20}$$

and

$$W_2 = \frac{1}{8}\{2\langle\hat{A}_1^\dagger\hat{A}_1\rangle + 2\langle\hat{A}_2^\dagger\hat{A}_2\rangle - \langle\hat{A}_1^2\rangle - \langle\hat{A}_1^{\dagger 2}\rangle - \langle\hat{A}_2^2\rangle - \langle\hat{A}_2^{\dagger 2}\rangle - 2(\langle\hat{A}_1\hat{A}_2\rangle + \langle\hat{A}_1^\dagger\hat{A}_2^\dagger\rangle - \langle\hat{A}_1^\dagger\hat{A}_2\rangle - \hat{A}_1\hat{A}_2^\dagger\rangle) + [(\langle\hat{A}_1 - \langle\hat{A}_1^\dagger\rangle) + (\langle\hat{A}_2\rangle - \langle\hat{A}_2^\dagger\rangle)]^2\} \tag{21}$$

By using Eq. (14), we can obtain the expectation values in the general form for the field operators $\hat{A}_1^{\dagger r_1}\hat{A}_1^{s_1}\hat{A}_2^{\dagger r_2}\hat{A}_2^{s_2}$ as follows:

$$\begin{aligned}
&\langle\hat{A}_1^{\dagger r_1}\hat{A}_1^{s_1}\hat{A}_2^{\dagger r_2}\hat{A}_2^{s_2}\rangle \\
&= |\alpha_1|^{r_1+s_1}|\alpha_2|^{r_2+s_2}\sum_{n_1,n_2=0}^{\infty} P_{n_1}P_{n_2} \\
&\times \left\{\exp[-i\lambda t(\gamma_{n_1+s_1+k_1,n_2+s_2+k_2} - \gamma_{n_1+r_1+k_1,n_2+r_2+k_2})]\cos^2\left(\frac{\theta}{2}\right)\right. \\
&\times \left[A_{n_1+s_1+k_1,n_2+s_2+k_2}A^*_{n_1+r_1+k_1,n_2+r_2+k_2} + \frac{n_1!}{(n_1+k_1)!}\frac{n_2!}{(n_2+k_2)!}\right. \\
&\times \left.\nu^2_{n_1+s_1+k_1,n_2+s_2+k_2}\nu^2_{n_1+r_1+k_1,n_2+r_2+k_2}R_{n_1+s_1+k_1,n_2+s_2+k_2}R_{n_1+r_1+k_1,n_2+r_2+k_2}\right] \\
&+ \exp[-i\lambda t(\gamma_{n_1+s_1,n_2+s_2} - \gamma_{n_1+r_1,n_2+r_2})]\sin^2\left(\frac{\theta}{2}\right)\left[A^*_{n_1+s_1,n_2+s_2}A_{n_1+r_1,n_2+r_2}\right. \\
&+ \left.\frac{n_1!}{(n_1-k_1)!}\frac{n_2!}{(n_2-k_2)!}R_{n_1+s_1,n_2+s_2}R_{n_1+r_1,n_2+r_2}\right] - \frac{i}{2}|\alpha_1|^{k_1}|\alpha_2|^{k_2}\sin\theta\, e^{-i\Omega} \\
&\times \exp[-i\lambda t\gamma_{n_1+s_1+k_1,n_2+s_2+k_2} - \gamma_{n_1+r_1+k_1,n_2+r_2+k_2})] \\
&\times \left[A^*_{n_1+r_1+k_1,n_2+r_2+k_2}R_{n_1+s_1+k_1,n_2+s_2+k_2} - \frac{n_1!}{(n_1+k_1)!}\frac{n_2!}{(n_2+k_2)!}\right. \\
&\times \left.\nu^2_{n_1+r_1+k_1,n_2+r_2+k_2}R_{n_1+r_1+k_1,n_2+r_2+k_2}A^*_{n_1+s_1+k_1,n_2+s_2+k_2}\right]
\end{aligned}$$

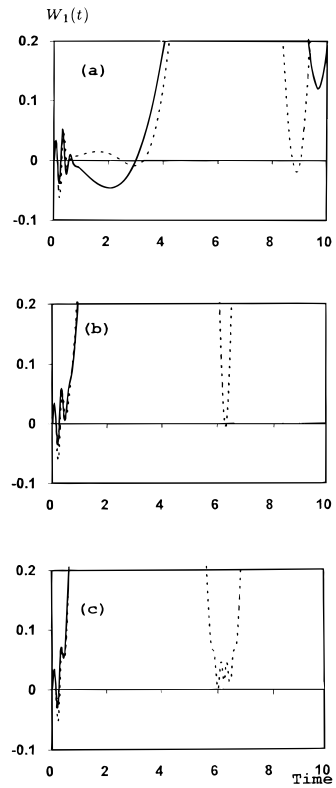

Fig. 1. Time evolution for the two-mode normal squeezing parameter W_1 of Eq. (20) with $\theta = 0$ (the particle initially in the excited state), $\bar{n}_1 = \bar{n}_2 = 10$, and the detuning parameter $\Delta/(2\lambda) = 0$ (dotted curve) and 5 (full curve) for various values of the parameters β_1 and β_2: (a) $\beta_1/\lambda = \beta_2/\lambda = 0$, (b) $\beta_1/\lambda = \beta_2/\lambda = 0.5$, and (c) $\beta_1/\lambda = \beta_2/\lambda = 1$.

$$
\begin{aligned}
&+ \frac{i}{2} \frac{\sin\theta}{|\alpha_1|^{k_1}|\alpha_2|^{k_2}} e^{i\Omega_{k_1,k_2}} \exp[-i\lambda t(\gamma_{n_1+s_1,n_2+s_2} - \gamma_{n_1+r_1,n_2+r_2})] \\
&\times \left[\frac{n_1!}{(n_1-k_1)!} \frac{n_2!}{(n_2-k_2)!} R_{n_1+r_1,n_2+r_2} A_{n_1+s_1,n_2+s_2} \right. \\
&\left. \left. - \nu^2_{n_1+s_1,n_2+s_2} R_{n_1+s_1,n_2+s_2} A_{n_1+r_1,n_2+r_2} \right] \right\}
\end{aligned}
$$

where $P_{n_j} = \exp(-\bar{n}_j)\, \bar{n}_j^{n_j}/n_j!$ for the coherent state and $\Omega_{k_1,k_2} = \phi - (k_1\psi_1 + k_2\psi_2)$ is the relative phase between the particle state (phase ϕ) and the field coherent state phase $(k_1\psi_1 + k_2\psi_2)$.

4. DISCUSSION AND CONCLUSIONS

Now we discuss the temporal behavior of W_1, which gives information on two-mode normal squeezing for two quanta $k_1 = k_2 = 1$, when we take $\bar{n}_1 = \bar{n}_2 = 10$ and different values of the angles θ and Ω with various values of β_1 and β_2 in the resonant and off-resonant cases.

Numerical results for Eq. (20) are presented in Figs. 1–8. Here we plot the two-mode normal squeezing W_1, Eq. (20), against λt in the interval [0–10] for $\bar{n}_1 = \bar{n}_2 = 10$ and different values of θ (namely 0, π/4, π/2, 3π/4, and π) and the relative phase $\Omega_{1,1}$ (namely 0, π/2) in the absence or presence of Stark shifts either in the resonant or the off-resonant case.

In Fig. 1 we display the results for $\theta = 0$ (the particle initially in the excited state) for $\Delta/(2\lambda) = 0$ (dotted curve) and $\Delta/(2\lambda) = 5$ (full curve) in the absence of the Stark shift, $\beta_1/\lambda = \beta_2/\lambda = 0$ (see Figs. 1a and 2a), and

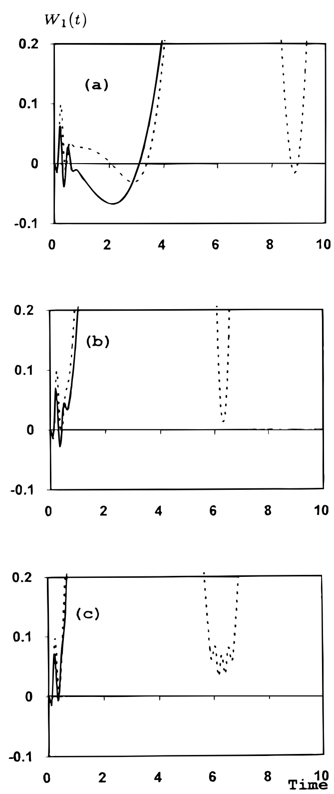

Fig. 2. The same as in Fig. 1, but for $\theta = \pi$ (the particle initially in the ground state).

in the presence of the Stark shift, $\beta_1/\lambda = \beta_2/\lambda = 0.5$ and 1 (see Figs. 1b–c and 2b–c), and in Fig. 2 we have the case for $\theta = \pi$ (the particle initially in the ground state) for $\Delta/(2\lambda) = 0$ (dotted curve) and $\Delta/(2\lambda) = 5$ (full curve).

Figures 1 and 2 clearly show in the absence of the Stark shift ($\beta_1/\lambda = \beta_2/\lambda = 0$) that the squeezing appears in three interval [0–1], [2–4], and [8.5–9.5] for $\Delta = 0$ (dotted curve) and appears in the interval [0–4] for the off-resonant case (full curve) (see Figs. 1a and 2a), while in the presence of the Stark shift, $\beta_1/\lambda = \beta_2/\lambda = 0.5$ and 1, the squeezing occurs in two intervals [0–1] and [6–7] for $\theta = 0$ (excited state) and also occurs in the interval [0–1] for $\theta = \pi$ (ground state) for two the resonant and off-resonant cases. Also, the amount of squeezing decreases with increasing β_1 and β_2. It is also

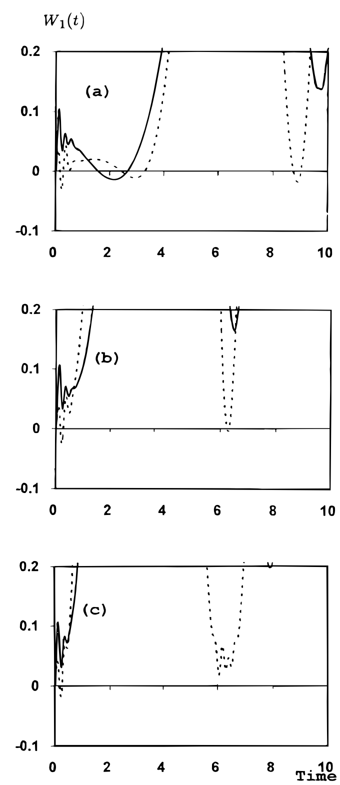

Fig. 3. Time evolution for the two-mode normal squeezing parameter W_1 of Eq. (20) with $\theta = \pi/4$, $\bar{n}_1 = \bar{n}_2 = 10$, the detuning parameter $\Delta/(2\lambda) = 0$ (dotted curve) and 5 (full curve), and the relative phase $\Omega_{1,1} = 0$ for various values of the parameter β_1 and β_2: (a) $\beta_1/\lambda = \beta_2/\lambda = 0$, (b) $\beta_1/\lambda = \beta_2/\lambda = 0.5$ and (c) $\beta_1/\lambda = \beta_2/\lambda = 1$.

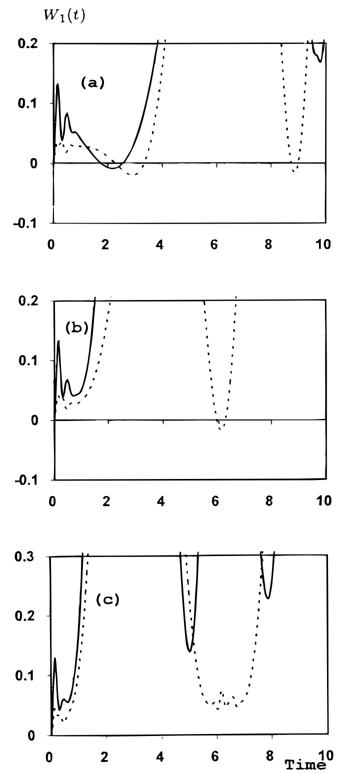

Fig. 4. The same as in Fig. 3, but with $\theta = \pi/2$.

apparent from the calculations that for these special cases, the phases do not affect squeezing.

In Figs. 3–5 we display the results for the relative phase $\Omega_{1,1} = 0$ and for different values of θ (namely $\pi/4$, $\pi/2$, and $3\pi/4$), respectively, in the absence and presence of the Stark shift.

In the absence of Stark shift ($\beta_1/\lambda = \beta_2/\lambda = 0$), we see that the squeezing occurs for $\theta = \pi/4$ in three intervals, [0–1], [2–4], and [8.5–9.5], while in the two cases $\theta = \pi/2, 3\pi/4$ the squeezing disappears in the interval [0–1] and appears only in the intervals [2–4] and [8.5–9.5] for $\Delta/(2\lambda) = 0$ (dotted curve) (see Figs. 3–5a) and, the squeezing also occurs in the interval [1–3] for the off-resonant case (full curve) in all the cases under consideration.

In the presence of the Stark shift ($\beta_1/\lambda = \beta_2/\lambda = 0.5$ and 1; see Figs. 3b–c, 4b–c and 5b–c) the squeezing occurs only in the resonant case $\Delta = 0$ (dotted curve). When $\beta_1/\lambda = \beta_2/\lambda = 0.5$ the squeezing appear twice for

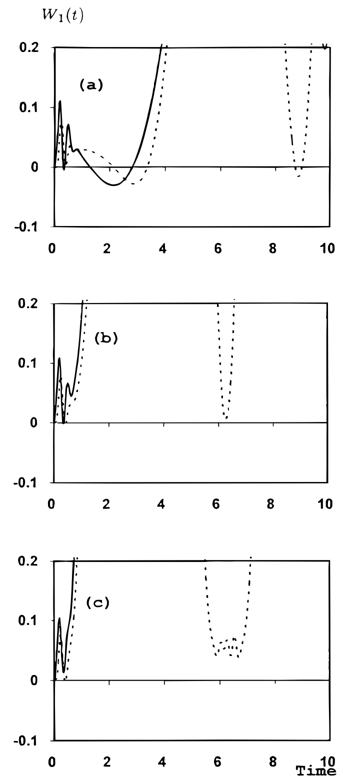

Fig. 5. The same as in Fig. 3, but with $\theta = 3\pi/4$.

$\theta = \pi/4$ and in the two cases $\theta = \pi/2$, $3\pi/4$ it appears only once, but for $\beta_1/\lambda = \beta_2/\lambda = 1$ the squeezing appears only once in the two cases $\theta = \pi/4$ and $\theta = 3\pi/4$ and no squeezing occurs in the case $\theta = \pi/2$, and also no squeezing appear in the off-resonant case, $\Delta/(2\lambda) = 5$ (full curve) in all the cases under consideration.

The same values in as in Figs. 3–5 are taken in Figs. 6–8, but with the relative phase $\Omega_{1,1} = \pi/2$; we observe from these figures that for the short time $0 \leq \lambda t < 1$ the maximum amount of squeezing is enhanced for $\theta = \pi/2$. We also observe that the squeezing in these cases (Figs. 6–8a) is larger than those in Figs. 3–5a. Furthermore, we notice that the squeezing occurs at later times for $2 < \lambda t < 4$ and $8.5 < \lambda t < 9.5$. For the first interval, by increasing θ, the two-mode normal squeezing increases, while the order is reversed for the latter interval. In the presence of the Stark shift ($\beta_1/\lambda = \beta_2/$

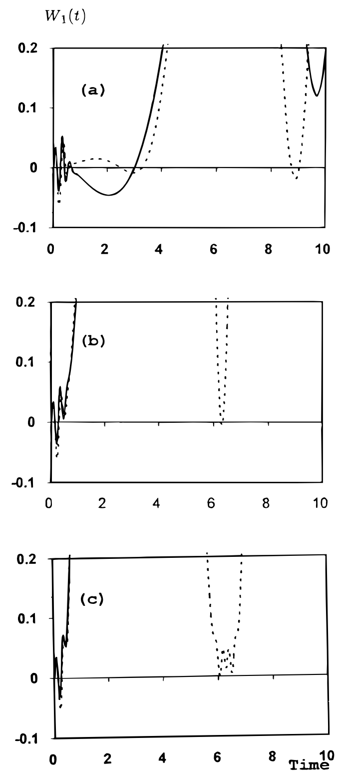

Fig. 6. Time evolution for the two-mode normal squeezing parameter W_1 of Eq. (20) with $\theta = \pi/4$, $\bar{n}_1 = \bar{n}_2 = 10$, the detuning parameter $\Delta/(2\lambda) = 0$ (dotted curve) and 5 (full curve), and the relative phase $\Omega_{1,1} = \pi/2$ for various values of the parameter β_1 and β_2: (a) $\beta_1/\lambda = \beta_2/\lambda = 0$, (b) $\beta_1/\lambda = \beta_2/\lambda = 0.5$, and (c) $\beta_1/\lambda = \beta_2/\lambda = 1$.

$\lambda = 0.5$ and 1; see Figs. 6b–c, 7b–c, and 8b–c), the squeezing occurs for all values of θ in the resonant and off-resonant cases.

The strong results of two-mode normal squeezing for the two cases (absence and presence of Stark shift) for the two-photon ($k_1 = k_2 = 1$) JCM is shown in Fig. 7a–c for $\theta = \pi/2$ and relative phase $\Omega_{1,1} = \pi/2$.

Numerical calculations show that for $\Omega_{1,1} = -\pi/2$, the behavior of two-mode normal squeezing is as observed in Figs. 6–8.

Thus we conclude that the effect of the relative phases on two-mode normal squeezing is the strongest for different values of θ in the two cases (presence and absence of the Stark shift) when the relative phase $\Omega_{1,1} = \pi/2$. In the presence of the Stark shift we have shown that the amount of squeezing decays with increase of the parameters β_1 and β_2.

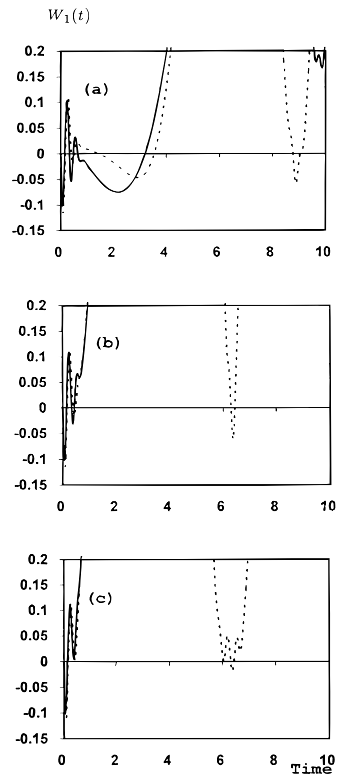

Fig. 7. The same as in Fig. 6, but with $\theta = \pi/2$.

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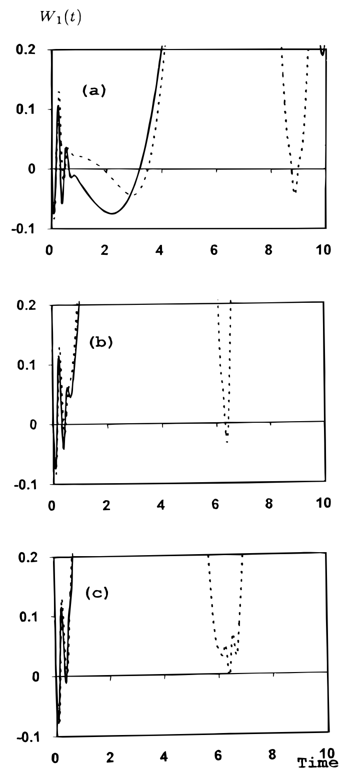

Fig. 8. The same as in Fig. 6, but with $\theta = 3\pi/4$.

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